Has Abstractionism Been Refuted?

1. Introduction

Abstractionism is the proposed explanation of the existence of abstract concepts and objects as creations of a human psychological process of 'abstraction', which is literally pushing one part of our experience into the foreground and giving it a name, while ignoring other aspects as irrelevant. The implicit theory is that all of our abstract concepts can be explained by repeated applications of this selection process, together with our imaginative capacity to combine concepts in various ways (as in Hume's example of a 'golden mountain'). For example, Keith Campbell writes (concerning the abstraction of tropes) that "An item is abstract if it is got before the mind by an act of abstraction, that is, by concentrating attention on some, but not all, of what is presented" (1981:§1). Lewis presents the idea more linguistically, as the giving of "incomplete descriptions" of concrete entities (1986:§1.7).

The theory originates in Aristotle's critique of Plato's theory of Forms. He wasn't prepared to accept the existence of abstracta as self-subsistent entities which we somehow directly 'perceived', and so he offered an introspective analysis of how we start from the given features of experience, and engage in a process of construction. In the middle ages this Aristotelian view became an orthodoxy, explored by Aquinas, and embraced by nominalists such as William of Ockham. It found particular favour with empiricists such as Locke and Mill (despite objections from Berkeley), who saw the possibility of explaining abstractions entirely in terms of experience. Arnauld and Nicole gave a particularly clear account of the theory, and H.H. Price has given the most sophisticated modern view.

This comfortable orthodoxy was shaken by the criticisms in Frege's *Grundlagen*. Frege was trying to define numbers, and thought abstractionism was hopeless. In his examples of a pair of boots, and a pack of playing cards, the number cannot be abstracted from the experience because more than one number is present. Frege had a general hostility to 'psychologism', as too subjective, and in his quest for a purely logical account of number he spotted equivalence classes, especially those created by one-to-one mapping. He offered a general account of abstraction by this means, with parallel lines giving us 'same direction' as his key example.

However, Frege was mainly concerned with numbers, and his rejection of abstractionism as a means of forming other abstract concepts was not clear-cut. Abstractionism survived as a view, and was at the centre of H.H. Price's 1953 account of thought in *Thinking and Experience*. At this point the debate became more vigorous, because Geach's 1957 *Mental Acts* used Fregean arguments to launch a fierce attack on Price, and on abstractionism of all kinds. Dummett endorsed the critique, and generalised it, and the old account has now not only fallen into neglect, but also into thorough disrepute. The consensus is that the Aristotelian account is hopelessly vague and misguided, leading to serious misunderstandings in logic, language and metaphysics. No account of thought that is based on introspection is much in favour any longer, because psychological and neuroscientific research has repeatedly demonstrated its epistemic unreliability.

Nowadays Dummett, Wright, Hale and Fine and others (with different motives) have firmed up the modern alternative view, based on equivalence classes. The standard formula is:

$$\forall\alpha\forall\beta(\S\alpha = \S\beta \leftrightarrow \alpha \approx \beta)$$

where α and β are expressions, $\S$ is an operator forming singular terms, and $\approx$ is equivalence, an 'equivalence' relation being one which is transitive, symmetric and (usually) reflexive. A relation between the referents of two separate expressions has resulted in a single abstract object, defined by an identity between the referents of the singular terms. Fine distinguishes 'conceptual' and 'objectual' versions, depending on whether 'α' and 'β' range over concepts or objects, and the procedure is often referred to as abstraction by 'recarving' (following a remark of Frege's), indicating that abstract concepts are generated by reorganising our current concepts in new ways, rather than creating concepts from scratch. The standard Fregean examples offered as illustration of the modern approach are that if α and β pick out two concepts seen to be equivalent by one-to-one mapping of their extensions, then these extensions have the same '**number of**' entities, and if α and β pick out two parallel lines, which are abstract objects, then the lines have the same '**direction**'. Another Fregean example is the derivation of same '**shape**' from the similarity relation between two triangles.

Dummett also offers (1991:173-5) an alternative view, expanding on Frege's earlier account (in *Begriffsschrift §9*), to explain the concept of 'suicide'. While the abstractionist might see 'suicide' as derived by singling out the feature of persons turning some implement of destruction fatally against themselves (ignoring, for example, which implements were employed), Dummett sees it in terms of linguistic substitutions. We start with the formula stating that 'person x killed person y', then note that possibly x = y, and then that in real life the name 'Cato' could be substituted for x, and finally that both instances of 'Cato' might be replaced with instances of 'Brutus', so "apprehending a pattern common" to the two sentences (p.174). The various ways to express what happened to Cato, some employing the word 'suicide' and others not, are all held to have the same 'content', while differing in 'sense' (p.175). Dummett takes this earlier account of logical abstraction to be a different way of presenting the same idea that is outlined in Frege's later discussion of 'direction' (1884:§64), but Hale disagrees (1997:§3), because Frege's earlier version is based on different parsings of a single sentence, where the later version depends on comparison of quite different sentences, and because the earlier version may generate new predicates, but it is hard to see how it would generate new singular terms. In this essay we will focus on the standard later version, particularly as the 'apprehending a pattern' of Dummett's version sounds a little too close to psychological abstractionism.

The modern view has clear advantages. Fine notes that "The prevalent view is that abstracts should just be treated as equivalence classes....The theory of abstraction thereby becomes a part of the much more comprehensive theory of sets or classes" (2002:2). The modern approach has played a key role in the development of modern neo-logicist accounts of arithmetic, where a second-order abstraction principle known as Hume's Principle is used to build a theory of the nature of numbers on the equivalence classes which were labelled 'the number of', generated by the relation of bijective mapping between members of two sets, which are then said to be 'equinumerous'.

The issue discussed in this paper is whether the abolition of old-fashioned abstractionism has unnecessarily impoverished our capacity to understand ontological issues, with insufficient gains in the modern approach to justify the impoverishment. The topic is huge, and so the main focus will be on the arguments which were used by Frege, Geach and Dummett against abstractionism, and the virtues of the older approach which we might still find of interest.

2. The critique of abstractionism

Berkeley objected to the whole idea of abstractions, on the empirical grounds that no matter how hard we try to think of some generalised abstraction such as a extension, colour or motion, it always cashes out as our experience of particulars. He first denies that he can form the generalised memories which are normally held to be the foundation of the abstraction process ("I have a faculty of imagining, or representing to myself, the idea of those particular things I have perceived....[but] I cannot by any effort of thought conceive the abstract idea" (1710:§10)), and he then claims that when we do use general words, which we patently do, the underlying images are an overlay of all the previous experiences ("a word becomes general by being made the sign, not of an abstract general idea, but of several particular ideas" (§11)). My own experience disagrees with Berkeley's (and, as we will see, agrees with Aristotle's), in that I have many memories of a highly generalised character, such as my memory of travelling by on motorways. Indeed, if we only had in our minds the many memories of actual expressions on a friend's face, it is hard to see how we would recognise them at next meeting, if their new expression were a previously unseen one. Berkeley concedes that we divide and combine particulars, in the way that Locke suggested, but denies that this process could ever reach abstraction. His approach is an anti-intellectualist one, in which these process are merely observed to happen within the mind, rather than being powers that we can bring to bear on our experiences. It is generally held that Berkeley's account is clouded by an excessively imagistic view of thought, so we will focus on later critics.

Frege's critique of abstraction arose out of his desire to define number: "to tell us how to abstract is not ...to give us a definition" (1884:§44). He became frustrated with the approach of abstracting objects down to 'units', which were being offered as the building blocks of arithmetic. The key difficulty was trying to abstract all individuality from the units, while somehow identifying each one as distinct. If, for example, you abstract from a pair of cats, hoping to arrive at the concept of 'two', then if you abstract from what distinguishes the cats, you arrive at the concept of 'cat', of which there is only 'one' (§34). He certainly didn't think we could abstract number directly from groups of objects. If a number is an answer to the question 'how many?', then zero and one must be numbers, but we can hardly discern the number one in the moon, and clearly there is nothing from which we can abstract zero. Since dogs have awareness of the world but never approach the actual concept of number, the numbers must be contributed by the "higher intellectual powers which distinguish men from brutes", rather than being abstracted from experience (§31). The abstractionist mistake is to think that we *experience* parts in relations to wholes, whereas if we realise that numbers apply as much to countable concepts as to countable objects, we see that there is actually a "logical subordination" involved (§9), such as the relationship between 'tyrannicide' and 'murder'. Thus we must shift from the psychological to the logical approach to abstraction.

If one aspires to derive number by direct abstraction from the world, these are powerful criticisms, and Frege presents a fairly conclusive case for saying that number is to be found in the world of concepts rather than in the world of physical objects. However, it is important to realise for present purposes that Frege was *not* rejecting the entire abstractionist account of concept-creation. Frege was only trying to arrive at a definition of *number*, and the abstractionist will have to find a subtler strategy if a rival account of number is to be offered. In the *Grundlagen* Frege was quite happy to accept abstractionism as a method (maybe even *the* method) of concept-formation. In §44 he supports the idea that we derive the concept of 'satellite' or 'object' from the moon, only becoming intransigent when 'one' is also thought to come to us in that way. In §48 he writes: "there is a temptation to suggest that we get the number by abstraction from the things. What we do actually get by such means is the concept, and in this we then discover the number".

Dummett argues that Frege eventually abandoned this view (1991:84-5), and spurned all abstractionist accounts in his later career, though Bell tries to make a case for this being a misreading (1994). In an unfinished review of Cantor of 1890-92 (1979:71) Frege writes that when you look at a great variety of books and "abstract from these properties which distinguish them, and thus arrive at, say, the concept 'book', this, when you come down to it, is no great feat", and he refers to the process as "ordinary abstraction". Even in his review of Husserl of 1894 (in which Dummett tells us that he "rejected abstraction altogether"), Frege seems happy to accept, in his sarcastic way, that "inattention is a very strong lye which must not be applied in too concentrated a form, so as not to dissolve everything", but this refers to his problems with arriving at units as a basis for number, and he does not directly reject 'inattention' as a means of normal concept-formation. The remark of Frege's which Dummett considers to clinch the matter ("in my opinion the bringing of an object under a concept is merely the recognition of a relation which previously already obtained" (1894:324)) is also in the context of Frege making his familiar point about the difficulty of deriving 'units' in this way, not in a discussion of general abstraction. However, when Bell quotes Frege from a 1906 article as writing: "we would

do well to be very careful about abstracting. But we should also not forget its beneficial effects", it appears that (contrary to Bell's view) Frege was once again being sarcastic, and that Dummett's reading is right.

So far a plausible picture is available, of concepts being derived psychologically, by ignoring and combining, but the process proving inadequate for the derivation of number, which concerns the concepts rather than their origin, and in need of much greater precision if we are to achieve an actual definition of number. The debate, however, shifted significantly with Geach's *Mental Acts* (1957). The target of Geach's strong critique of *all* abstractionist accounts of concept-formation was H.H. Price's *Thinking and Experience* (1953), and Geach expands Frege's hostile attitude to cover concepts such as 'red', 'big' and 'left and right', as well as (more plausibly) numbers and the logical connectives.

We can divide Geach's arguments into two groups, the Fregean ones which are aimed at abstractionist accounts of number, and by extension to logical connectives, and Geach's new arguments against the whole idea of concept-formation by abstraction. The first group reiterate Frege's views, giving examples such as our inability to see what number might be abstracted from a rhyming couplet, making the claim that we must have the ability to count *before* we can bring numbers to bear on the world (1957:§8), and attacking any psychological account of logical connectives, because of the subjective instability which would ensue in logic (§7).

The first of Geach's more general arguments against abstractionism is that while our immediate experiences of the physical world, such as yellowness or wetness, may be available for abstraction, they could never get us to the underlying reality of actual substances such as gold or water (§6). He calls abstractionist approach 'claptrap', but he appears to have a rather simplistic view of what is being claimed. The problem of asserting truths about entities outside our phenomenal experience is an epistemological difficulty for any empiricist view, but empirical epistemology has an armoury of tools for extending our experience, such as induction, probability, implication and best explanation. If abstraction of actually experienced yellowness is indeed possible, this could only happen by holding numerous experiences of yellow in consciousness. Abstraction (according to Aristotle) occurs in the mind, not in the sense organs. The abstractionist route to the atomic number of gold starts with its observed colour, weight and malleability, and moves through its chemical behaviour by a series of large abstractions to the periodic table, and then back to gold's location in that framework. Each step can plausibly be achieved by holding these layers of information in consciousness, and then ignoring what is irrelevant and focusing on what seems germane (a hallmark of general intelligence). Other scientific 'substances' such as quarks are derived by picking out patterns of physical behaviour, especially those that fit with our mathematical accounts of physics. Geach wants to emphasise the way in which our concepts are 'created' rather than 'derived', but one can hardly create the concept of a quark as if one were writing a poem. There doesn't therefore seem to be any bar to arriving at a concept such as 'substance' by the abstractionist route.

Another area of our experience which he feels would never yield the appropriate concepts by abstractionism is introspective psychology (§6), his reason being that concepts such as 'judging' or 'desiring' emerge *directly*, rather than being abstracted by some introspective sixth sense, analogous to our abstracting yellow from visual experiences. His argument (which he considers 'conclusive') is that we could not form an analogy between a psychological characteristic and a sense characteristic if we did not have a prior grasp of the former. Again Geach's attack is rather simplistic, involving a common misapprehension in discussions of empirical concept-creation (such as the concept of 'causation') – that of seeing root experience as largely *passive*. He pictures us searching introspectively for mental objects such as desires, and then applying the abstractionist process to reach a concept of them, as if we were categorising butterflies. But desires become evident to us when we observe patterns in the behaviour of ourselves and others (including animals). If one compares an animal's desire for drink, a baby's desire for a toy, and my desire for a degree, to focus on some aspects and ignore others quickly reveals all three to be in some respect identical. It seems an obvious step to label this common respect, and then attach it to the way I feel when I yearn for my qualification. There is no question-begging necessity for me to *already* be able to label the psychological state of 'desire'. It seems that there are no more impediments to the abstraction of psychological concepts than there are to the abstraction of the concepts of the properties of physical objects.

A particularly puzzling objection offered by Geach is that the concept 'big' could not be attained by abstraction, because whether something is big is a relative matter (§9). This objection, more than the others, shows how grossly simplistic is Geach's view of his opponents' proposals (and he stuck to the point when he revisited the topic 26 years later, calling the proposal 'an absurdity'). In his 1983 account (p.168) he will not allow that Minnie can be seen as big *for a mouse*, because Minnie is not big *for an animal*, so (on the abstractionist account) "Minnie cannot be considered as big or small". The simplification, which undermines Geach's whole case against abstractionism, is that he considers each case in isolation. Plainly 'big' is a relative concept, so if you meet some token of a completely unfamiliar type for the first time, you can have no idea of whether it is big, and there would be nothing to abstract (though it would be easy to abstract the new object's 'size', if you wanted to fit it through a particular aperture). One could hardly, though, fail to spot one very bulky mouse among a hundred skinny ones, and just as easily spot a bulky elephant among some skinny elephants.

What possible objection can there be to noticing that the two experiences of bulk-detection have something in common? It is only requisite that we have made a prior set of abstractions, to a generalised picture of what a normal mouse or elephant should be. A key issue that begins to emerge is whether abstraction should be understand as an automatic, or intuitive or intellectual process. If it is simply automatic, then a 'second-level' abstraction would be hard to explain, as it isn't a direct response to a sense-experience, but if (as one should) one allows an intellectual component, then very refined abstractions at several moves from experience should be no surprise. 'Abstract' in normal English usage means removed from concrete experience, and it comes in degrees. The history of recent

mathematics is usually described as one of increasing abstraction, as the generalisations become ever broader, and the rules become increasingly generic. So a relative concept such as 'big' seems to emerge quite straightforwardly from an abstractionist account of concept-formation.

A more plausible problem case invoked by Geach is that of two interdependent abstract concepts, such as 'left' and 'right' (1957:§9), or 'striking' and 'being struck' (1983:168). His complaint is that a single act of abstraction should only give rise to one half of the concept, with a quite different explanation required for its completion. Geach complains that because the two concepts of 'striking' and 'being struck' are quite distinct, they ought to be abstracted from *different* events; the fact that they are two ways of seeing the *same* event is held to prove that we impose the concepts on the event, rather than the event generating the concepts. But an obvious response is that what is initially observed in an incident of striking is a *relational* event, suggesting an initial concept of 'x-strikes-y', and that the active and passive forms can then be derived from this relational concept.

For example, Quine offers us two ways of doing this (1960:344), either by forming the predication 'x strikes y' (Sxy), and then applying an existence prefix to get 'something y is such that x strikes it', or by a reverse procedure which he dubs 'derelativization' (a procedure, incidentally, of which Geach made great use elsewhere in his writings), which uses an operator to create a new predicate 'bites something', which is then used for the predication 'x bites something'. The point of the strategy is that a two-place relational predicate can always be re-written in one-place form, and so our concepts of 'striking' and of 'being struck' could be generated by two operations of Quine's procedures, applied to the single relational concept abstracted from experience.

The mention of 'experience' reminds us that abstractionism is an account which is largely favoured by empiricists, that striking and being struck are also two different experiences, and that we should expect different concepts to arise from some event if it can be experienced in different ways, as we find with 'pig' if you rear the animal, and 'pork' if you eat it. Even a dog would be expected to have some sort of distinct notions of biting and being bitten, as two very different experiences in an obvious close relationship.

Left-and-right is a subtler case, but again the concepts seem to arise from immediate physical experience. Geach is struck by the difficult *logic* of how the two relations are connected, but there is no difficulty for even the most uneducated of human beings in grasping the asymmetry of being left-handed, or deaf in the right ear, and thus in grasping one half (or the other) of the concept. We can again account for the parts of the concept with a derelativization procedure which starts from an abstracted relational concept of 'left-and-right', and generates the two necessary one-place predicates. The experience of 'up' and of 'down' are similarly indissolubly related for creatures in a world of gravity, and many of our experiences confront us directly with pairs of related opposites, such as hungry/sated, alive/dead or love/hate. Any further difficulties with unravelling the connection of left and right comes from their indexical origin, rather than from any peculiarities of space, or from originating separately.

Perhaps Geach's favourite example is the fact that the concepts of 'red' and of 'chromatic colour' can be rooted in an identical experience, and yet are quite distinct concepts, so that no direct abstraction from visual experience could ever explain the distinction (1957:§10; 1983:171-2). This seems to indicate that not only does Geach fail to see that abstraction is built on our experiences of events, rather than on the events themselves, but he also seems reluctant to address crucial elements in the abstractionist picture – that not only can the initial abstraction be of a complex concept rather than a simple one, but also that repeated abstractions can produce second- and third-level abstract concepts, with dependence relations between them. Thus Geach presents 'chromatic colour' as if it were a quite distinct concept that might absurdly be derived directly from a red object, but it seems obvious that 'chromatic colour' requires a prior grasp of 'colour', and that the notion of a 'colour' is unlikely to be graspable if one could not first identify distinctions between actual colours such as 'red' and 'blue'. The only way to achieve clarity seems to be through a consideration of the psychological starting point, of experiencing various precise shades of a colour such as red, often in the context of the red gradually changing its shade (in sunsets, rainbows and apples). We experience the shades of red as a single experience, which can be both analysed and synthesised, and then treated in our language in the normal way (such as with qualifiers like 'roughly'). An obvious hierarchy emerges here (routinely employed when one refers to 'different levels of abstraction'), which might be developed in this case by starting with the concept of 'red', analysing *down* to the more precise shades of 'vermillion' and 'scarlet', and coalescing concepts *upwards* to attain 'coloured', and then 'brightly coloured' or 'chromatically coloured', continuing the process to talk of the 'qualities' or 'aspects' of objects, and subsequently writing papers about the concept of a 'property'.. Philosophers of mind have increasingly identified 'meta-thought' (the capacity to think about one's own thoughts) as what is distinctive of the human mind, and even what actually constitutes consciousness, so any understanding we have of our concept formation needs to fit into that picture.

There is not much doubt that the concepts of 'number' and of the logical connectives are the most challenging for an abstractionist account of the existence of abstract concepts, and Geach makes the Fregean case very effectively – that they are pure, stable and objective, in a way which is at odds the messy subjective origins proposed by abstractionism. To Frege's question about the number that we might experience in viewing a pair of boots or a pack of playing cards, Geach adds his own example of the many numbers to be discovered if we examine a rhyming couplet (1957:§8). Nevertheless, it seems unlikely that we would have any interest in such 'pure' abstractions if they in no way addressed facets of our experience. Geach's claim (§8) that "the recitation of successive numerals...is logically and temporally prior to counting objects" is not convincing, given that many species of animal have a clear sense of numerical quantity. Common sense suggests that the ability to imagine the increase or decrease of some collection of objects precedes in every way the ability to denominate such a continuation by symbolic means. If the earlier point about the interrelatedness of 'up' and 'down' in our experience is accepted, then it would take very little

abstraction from experience to arrive at the concepts of disjunction and negation (despite Geach's assertion (§7) that "in the sensible world you would find no specimens of alternativeness and negativeness", when such things would seem to be the very stuff of conscious existence). Geach claims that his own experience of 'or' is in the threat '..or else!' (§7), so abstractionist should apparently derive the concept of a threat from the experience, but this neglects the role of linguistic convention in the abstractionist account. The basic abstractions of ordinary language could hardly be the quirky derivations of individuals, but must emerge from a consensus of widespread usage, founded in a shared life. If the abstractionist story is accepted, then we can historically reconstruct (in outline) the origin of a concept from its basic meaning, and clearly threats have nothing to do with the origin of our shared concept of disjunction.

Dummett shares the dismissive view of abstractionism which Geach asserts with such force. Two further interesting Fregean points are added by Dummett to Geach's criticisms: a general rejection of all 'genetic' accounts of concepts (by pointless speculation about their origins), and a flat rejection of the involvement of mental images in the matter. The first point rests on a challenge about conceptual priority: "if it were impossible to say in what the possession of a certain concept consisted, then there could hardly be a start to the enquiry by what means it was acquired" (1973:677). One might reverse this argument, and ask whether we could ever entertain concepts the acquisition of which was inconceivable to us. Though people find their heads full of well-developed concepts long before the notion of their origin occurs to them, in reality it is obvious that origin will precede existence. Dummett's point is, of course, well made, that an enquiry must start from the intrinsic character of the concept, which is immediately known and available for analysis, while the origin of the concept is speculative and shrouded in uncertainty. But the main contention of this paper is that if you wish to study their ontological status, then the origin of our concepts (however difficult to articulate with precision) becomes exceedingly important.

The other point Dummett makes (principally against Berkeley's account) is that "it was left to Frege to perceive that the sense of a word has, intrinsically, nothing to do with mental images whatever" (1973:159). It is very difficult to evaluate this assertion by means of introspection, and if one approaches the notion of sense from the point of view of possible inferences, then it certainly seems that mental images have nothing to do with the matter. If, however, we approach the question by the discredited route of the *origin* of a word's sense, then some discussion of images seems unavoidable, especially when the concepts might be regarded as generalisations about visual experience. Inferences about triangles have no need of diagrams, and introspection of our concept of a triangle reveals an oddly blurred experience, somewhere between Aristotle's generalised image and Berkeley's multifarious ones. But the idea that we might arrive at the concept of a triangle without the involvement of some sort of image, or at least the ability to *draw* a diagram, seems inconceivable. The fact that we can now express the nature of a triangle as a set of equations is precisely the sort of result of a series of steps away from the original images which the abstractionist account would predict.

Thus we have seen that a group of arguments which have become a settled orthodoxy in recent years are actually open to quite a strong challenge. Critics of Aristotelian abstractionism are forced to deny (or play down) features of mental life which not only could contribute to abstract concept-formation, but must almost inevitably have such a result. The main defence of the older view is the claim that it has been misunderstood and oversimplified by its critics. Frege retained a lifelong focus on the question of number, despite apparently losing faith in the abstractionist approach to concepts, but it seems that no wholesale criticism of abstractionism predates the publication of Geach's book, and that total dismissal of the process is a quite recent fashion in philosophy.

Six claims emerge about the nature of the misunderstanding (or distortion) which has occurred: **first**, that while logical considerations are important, and dominate modern analytical philosophy, they cannot oust all other forms of explanation, including those which involve introspection and origins; **second**, that abstractionism is not a uniform process, but can be seen in three guises, as a non-conscious associationist process, as an act of intuition, and as a fully conscious and rational process; **third**, that abstraction takes place not from the world, but from our experience of it (as Locke regularly insists), so that concepts such as 'big' can have relative meaning, despite referring to a single sort of event in the world; **fourth**, that several concepts may be derived from a single experience, as when a relational action gives rise to passive and active forms, or indexical aspects; **fifth**, that abstraction can be iterative, proceeding in stages, so that remote abstractions which seem unreachable by a single psychological act can be straightforwardly accounted for as a third or fourth step in a complex process (a point reinforced by the widespread ordinary English usage of the word 'abstract', which is felt to be on a scale from 'a bit abstract' to 'highly, or ridiculously, abstract'); and **sixth**, that when new abstractions emerge into ordinary language, they are stabilised and standardised by daily usage in the community, so that there is virtually no room for the highly subjective and solipsistic creation of quirky concepts which so bothers the critics.

This is not to negate the force of some important criticisms. The great virtue of the modern view is its aspiration to clarity, precision and symbolic expression, and Frege's approach to metaphysics from the direction of mathematics rather than from daily life has rightly been very influential. We are now, though, at the point where the role of pragmatics, context, social convention and non-linguistic thought must be brought into the picture, and here it seems that the way in which abstract concepts are formulated must receive more attention.

3. Restoring Some Balance

Whatever we may think of the shortcomings of abstractionism, we must now consider reasons why the approach through equivalence principles is not adequate to replace it. There is a drift towards the assumption that the latter approach can now do the whole job, as when Hale, having referred to Hume's Principle, says that "I shall suppose that abstract objects may always be reached in this way" (1987:67). The modern account of abstraction shows obvious

parallels with the discussion of truth in the first half of the twentieth century. Debates over pragmatic, correspondence and coherence accounts seemed sterile, and led to a drastic proposal that the whole concept of truth was redundant. Only when Tarski offered a rigorous definition of 'truth' in terms of 'satisfaction' did the concept once again take a prominent role in philosophy. But fans of a more 'robust' account of truth remain frustrated by the narrowness of the Tarskian approach, and there appears to be a similar situation in recent times with the question of abstraction.

The account of abstraction by 'recarving' our present store of concepts meets an obvious difficulty if we ask how the whole process gets started. If the idea is that the concept of 'redness' arrives through an identity between concepts which is fixed by the equivalence of objects which have the property of being red, then we must first acquire the concept of 'red' as a property of the things concerned (before we 'abstract' it). It is hard to see how the equivalence formula could do the initial job, since some common feature of reality must first be observed. Additionally, we need functions, relations, implications and terms to build our basic formula, so some means of deriving those concepts would also be needed. The question of how this whole assemblage of abstract concepts gets off the ground in practice has become a taboo subject.

As befits a product of analytical philosophy, the modern approach to abstraction uses symbolic formulae, precise definitions and sharply delimited classes, but this means that the philosophers have cut themselves off from studies of animal behaviour and infant psychology. If abstractions are to be understood entirely in terms of the abstraction principle, then there is a strong temptation to assert a priori that animals lack concepts (a temptation to which Geach succumbed in 1957, denying all real concepts to the "brutes", though he relented a little in 1983 (p.160)). But the modern evidence is overwhelming that many animals can follow deductive steps, and respond to situations in which cardinality is the only significant fact. A full account of abstract concepts must include a non-linguistic dimension, even if we concede that they only have sufficient clarity and precision for useful study when they take symbolic form.

An interesting feature of the modern abstraction principle is that it seems to require precisely two entities for its operation. We can say that the number of Fs is equal to the number of Gs (where F and G are concepts), and proceed to obtain the full equinumerous class by repeated applications of the principle (e.g. the knives-and-forks, then the forks-and-spoons, then the spoons-and-plates). An equivalence class can exist with only one member, but two objects or concepts are needed to operate the principle which generates the class. There are plenty of cases which fail to fit comfortably into this account. Some entities can be thought of in abstraction which are self-evidently unique, such as Gordon Brown's smile, to which nothing is equivalent. The equivalence principle might be a route to the creation of the concept of a 'smile', based on equivalent facial expressions, but the smile under consideration is unique, and can be discussed in abstraction. The concept of an object's 'centre of mass' seems to be an obvious abstraction, but each centre of mass is different, not equivalently related to other centres of mass. Rosen calls this inability of the equivalence principle to produce some of our abstract concepts the problem of 'undergeneration' (2006:8). It has been noted, for example, that the concept of a 'set' is the quintessential abstract concept in modern analytical thought, but sets are defined purely by extension, not by an equivalence relation (and especially not the empty set).

A related difficulty arises with concepts that need *more* than two entities to get off the ground, such as being the 'odd-one-out' (or being a 'big' mouse). We might form an equivalence class from many clusters of things, each cluster having a single misfit member, but we seem to need the concept of 'misfit' to arrive at that point, and common usage tells us that we could never refer to an entity as a 'misfit' if only two such items existed (could one of the two poems of Homer be described as a 'misfit' among his works?). Some abstract concepts seem to exist as part of a substantial interdependent cluster, such as the basic notions of Newtonian physics.

Another odd feature of the modern view is that, in the pursuit of objectivity and rigour, the creation of abstract concepts has been taken out of human control. If two entities are found to be in an equivalence relation, then we are faced with a new 'recarved' abstraction whether we like it or not. This takes the general picture of the world of abstractions a very long way from the original phenomenon which we were trying to describe and explain. For example, there is an equivalence class of all the people in the world who have three brown buttons on their coat, with just one of them undone. There must therefore, it seems, be an abstract concept that belongs with this class, even though no language is likely to have a word for it, and speakers would probably reject the concept if philosophers offered it to them. Because the rules generate abstract concepts beyond human control, Fine has to devote a great deal of his book on the subject (2002) to dealing with the problem of their 'hyperinflation', which is not a problem if the matter is approached in a more human and less logical way.

There is a parallel difficulty with those abstract concepts which would *not* be permitted, on grounds of vagueness of the property or uncertainty in the resemblance. The attraction of the equivalence class approach is the precision of the partition between concepts which is generated, but this depends on a perfect resemblance between the properties which lead to the equivalence. If two things resemble one another because they have an identical property ('pillar box red'), this will generate the necessary transitivity in the relation, but any vagueness in the property (such as 'reddish') will lead to a degeneration in the transitivity, and a collapse of the equivalence. An imperfect resemblance or mere degree of similarity, which seems to provide us with many everyday concepts (such as 'scruffy' or 'light-coloured'), cannot even get started in the modern account. There is certainly no prospect of Geach being able to derive our shifting concept of 'big' by such means.

At the heart of these difficulties seems to be the question of whether the identification of an equivalence class can give us the sense as well as the reference of the new abstract concept. If the concept of 'red' required no more than the picking out of the class of all ripe tomatoes, pillar boxes etc., then reference to that class could be achieved in all manner of ways, such as 'class number 467', or 'the class it is taboo to name'. But 'red' must play innumerable roles

in thought and speech, in discussions of counterfactual situations, resemblances between abstractions, metaphor, generalisation, and so on. A blind person can apply the abstraction principle to name the set of red things, but could hardly have a full grasp of the abstract concept of 'redness'. Even if we allow the existence of non-verbal abstract concepts (in adult humans, as well as in infants and animals), they will need this generic character if they are to play a role in wide-ranging thought, even of an imagistic kind. Clearly we need to know (in Frege's later terminology) the *sense* of 'red', as well as the *reference* to the relevant group of items. But how can that emerge from the equivalence principle?

Finally there is the important matter of the ontology underlying the equivalence principle. We are invited to see the process as 'recarving' the set of concepts which we apply to a given state of affairs. The aim is that the state of affairs is untouched by the logical manoeuvres, and so reality is in no way augmented, but at the same time the procedure generates new singular terms with identity conditions, and the Fregean approach is to say that a singular term which has a role in a true sentence must make reference to an existent object over which we can quantify. Thus our state of affairs acquires no new objects, but our discourse does. The approach is not surprising in a platonist like Frege, who wanted a huge proliferation of abstract objects to support his account of vast cardinalities. The platonist followers of Frege are complacent about this rather strange state of affairs (in which a straightforward concrete world is accompanied by a world of abstract objects whose extent boggles the human imagination, most of them beyond human grasp because they are logically generated by precise resemblances which are too complex or vast for our puny intellects), but if one explanation of abstraction delivers 'objects' (of a modern Fregean character), and the other merely delivers psychological 'concepts', there are good ontological reasons for being sympathetic to the latter. Hale (1997:§5) claims that there is no further ontological commitment to be found when a given state of affairs is recarved, but his 1987 book has an extensive defence of the ontological commitment to objects whenever there is a singular term in a true sentence (as long as it meets his condition of not being easily paraphrased away). In the case of a number, a "self-subsistent object" is required (Hale and Wright 2001:14). To generate numbers as self-subsistent objects by means of equivalence, while avoiding adding new objects to each state of affairs, is a tricky compromise which supporters of the modern view find hard to achieve.

We have already seen that the critical rejection of the older view of abstraction involved some significant misunderstandings, and we have now added some serious limitations in the modern view which has replaced it. But the thesis of this paper is that this 'replacement' (or elimination) of the older view is simply the result of an over-enthusiastic attack by Geach. No one can object to the interesting explorations of the Fregean approach to abstraction, which have found a place for it in logic and set theory. We have now seen, though, that there are large areas of abstract thinking (in humans and animals) which cannot be supported by the Fregean view, while huge swathes of abstract concepts are generated where no one wants them, and (above all) the modern view doesn't seem to explain or clarifying our *understanding* of abstract concepts, which is the essential requirement for their use in normal speech. Tait's remark (1996:41) that we have "inherited a poor vision of what philosophy is" from Frege seems a bit ungrateful, but some widening of our view certainly seems to be called for. With this in mind, we must return to the older view, to see whether it has the resources to meet some of our theoretical and metaphysical needs, and hence to deserve a role in modern debate, rather than the status of a refuted and discredited theory.

4. Abstractionism reappraised

We will now look in more detail at the virtues and possibilities of abstractionism. Aristotle, who originated the psychological approach, tried to identify features of our thought which might give rise to them, his most striking observation being that "memories which are many in number form a single experience" (Post. An. 100a5). This seems to be correct (despite Berkeley's denial), and invites an introspective inventory of other relevant mental features. Aristotle was happy to say that we experience wholes, followed by attention to parts (Phys 184a22), but he was less happy that we could psychologically abstract properties from particulars (Phys 193b36). In a reversal of Frege's view, he thought it was much easier to abstract down to mathematical units (Phys 193b33, Met 1061a, Met 1078a). Arnauld and Nicole disagree with Aristotle when they claim that the mind begins with the parts rather than the whole, and we can only know things if the mind "considers them a part at a time" (1662:1.5). They give the nice example of a footpath, where we can focus on its length, or its width, or its direction, this single object coming to our awareness by means of its parts, as a result of human need. Whether we take the whole or the part as the starting point, the immediate perception of part/whole relationships is one aspect of what we can call 'The Given'.

Labelling these introspective findings as 'The Given' invites a comparison with work of Sellars and McDowell on the a difficult question of whether abstract concepts enter our experience unbidden at the very earliest stages of perception, or whether there is some 'raw' data which is then conceptualised. In the context of abstractionism, the issue emerges with the question of the role of the intellect in the abstraction process. The early debate (e.g. Locke 1690:II.xxii.2) leaves us with a standard tripartite picture: that there is a degree of abstraction which arises quite mechanically (seen most clearly in Hume's notion of 'resemblance'), then a level of creation which might be called 'intuitive', being conscious but fairly unmediated and unreasoned, and finally a level of intellectual choice, where items can be selected and combined as part of a fully conscious rational strategy. A quite mechanical abstractionism could plausibly be attributed to animal minds; the intuitive level has the role of creating the abstractions which form the bedrock of normal speech (such as properties, number, logical connectives); rational abstractionism is a more creative process, producing the notions found in more sophisticated cultural areas.

An appeal can also be made, as part of this 'Given', to the immediate perception of archetypes, or of what is typical in a kind. A perception of the type, founded on resemblance, must precede the perception of the archetype. Natural

kinds are a particular case of this perception, as when the *Upanishads* tell us (in 'Chandogya') that "By knowing one lump of clay, all things made of clay are known; by knowing a nugget of gold, all things made of gold are known". From an evolutionary perspective, the perception of kinds by even the most primitive living creatures would seem to be a biological necessity. Fine raises a difficult question for abstractionism here (1998 §5), of why a property which has been isolated by psychological abstraction should then be regarded as a *type*. This is a difficult question for any account of the matter, but the remark from the *Upanishads* suggests that the idea that clay is a type might arise fairly mechanically from pooled experience over time. We might also describe this facet of The Given as mechanical or intuitive generalisation, which some have considered to be the most basic component of abstractionism. Russell calls such processes "a condensation of many inductions" (1940:76), which he holds would give rise to the word 'dog', reminding us that a spectrum ranging from mechanical 'habit' to rational 'inference' is found in the Humean debate about induction, just as we are finding in The Given that precedes abstraction.

Among other ingredients of The Given, the phenomenon of mental 'focus' is self-evident, and such focusing is found both in perception and in thought. In the sentence 'Your desk, apart from its colour, fits my room perfectly', it is even plausible that linguistic reference needs some sort of notion of focus if it is to be fully explained. The presence of the fovea in the retina demonstrates the biological basis of the phenomenon, and the much cited (and derided) feature of 'ignoring' aspects of experience is a necessary concomitant of the ability to focus. The case in favour of abstractionism here is that the fact that conscious experience is in a state of constant focus-and-ignoring is not just a convenient tool that might be used to abstract, but that such psychological abstraction is quite inevitable and unavoidable, whatever logical structure one may then discern in the results. Mechanical abstraction will happen in any creature which experiences the world, and intellectual abstraction is an available strategy when we contemplate the world rationally.

In Price's account, the phenomenon of 'recognition' is taken as the founding feature in the psychology of the abstraction process. He rejects the intellectualist view (1953:53), noting that animals must be able to recognise their food, such as pheasants recognising blackberries. Fundamental to this, he suggests, is the ability of even an animal to recognise many geometric shapes, even though they are too complex for humans to conceive or imagine (with such precision). This recognition occurs despite each instance of it (each blackberry) varying a little from any previous experience of the same type.

The full picture of the visual aspect of The Given is completed by the ability to imagine, and the ability to combine, so that our simple abstracted concepts, of shape, for example, can be assembled imaginatively into abstract objects of tremendous complexity, such as a monster in a computer game. More subtly, and controversially, we may cite among aspects of given experience which participate in the abstractionist process such phenomena as dilemma, connection, absence, incompatibility, magnitude, possibility, inevitability, and expectation. These are clearly universal features of conscious experience, and so if abstractionism is an *inevitable* process, we can ask (in a state of innocence) which abstract concepts might be expected to arise from them.

The next part of the traditional account offers stages of abstraction which are built from what is psychologically given. None of these accounts, of course, attain to the precision sought by modern logicians, but a degree of consensus is found in the picture, and some of it is amenable to empirical research. Kant gives the neatest picture of the steps of abstraction (quoted from *Jäsche Logic* of 1800 by Bell, 1994:153): first, *comparison* of representations in relation to the unity of consciousness; second, *reflection* on how the representations can be conceived in one consciousness; and finally, *abstraction* of everything in which the representations differ. This is a highly intellectualist account of the matter, in which animals are unlikely to participate. It is worth noting that Kant's final step is not very distant from the Fregean formula, in which the aspect which generates equivalence is selected as the basis of the new abstract concept.

Price is more aware of animal minds in his account, and offers the idea of "recognition by signs" (1953:75) as a barely conscious step which follows simple recognition, his example being the non-conceptual recognition of ice as cold, in the instant wake of the recognition of the ice. We might re-express Price's thought as the proposal that prior to any abstraction process it is essential to recognise both the relevant objects (or stuff) and also their properties, although this will inevitably invite a question about the psychology which underlies the metaphysical difficulties concerning whether we understand objects as bundles of properties (implying the priority of the properties), or as a substance with accidental and essential properties (where the recognition of the substance seems to be the initiating step).

It is not a requirement of the abstractionist view that the metaphysical structure of our conceptual scheme should be identified with the order of steps in the creation of its abstract components; it is merely suggested that the metaphysical picture will be more comprehensive if such thing are kept in mind. Trope theorists, for example, like abstractionism, since an object is held to be comprised of abstract particulars, and we must presumably be able to pick these out from their hosts if we are to discuss them (Campbell 1981:126). Perfect resemblance between the tropes is then held to create equivalence classes, which play the role of universals. Attending to the psychological steps in the creation of trope-concepts draws our attention to difficulties of circularity in the trope account, since recognition of a trope as being of a certain type seems to precede picking out other tropes as candidates for the necessary resemblance. This is no knock-down objection to trope theory, since every account of such things faces circularity difficulties, but trying to clarify the psychology is a step towards the correct account.

Having acquired a picture of the psychological Given, and then the possible steps in the abstraction process, we arrive at what may be the greatest strength of the abstractionist view – its ability to give some explanation of 'levels' of abstraction. Any philosophical account of a concept must be responsive to the ordinary language usage of relevant

terms, and sophisticated discussions in ordinary English take for granted the picture of ideas being more or less abstract, as in the idea that (say) economic inflation might be discussed at a more abstract or less abstract level. The most evident examples, in the abstractionist account, arise from repeated applications of the process, as when we refer to 'mugs' (ignoring colour and minor variants in shape), then to 'drinking vessels' (ignoring more aspects of the shape, but not all), then to 'crockery', then to 'household objects', then to 'physical objects', then to 'stuff'. A commonplace hierarchy is invoked, in which everybody knows that ultimately mugs are a sort of stuff. It is, of course, open to the equivalence principle theorists (e.g. Hale 1987:61) to invoke a hierarchy of abstraction by repeated application of the formula, but we have seen some problems with that, and more will be raised below.

Aquinas observes that levels can arise from the *source* of the abstraction, as well as from the progressive remoteness from the starting point (*Summa Theologica* §85:Ad.2). His account (which also contributes to our picture of The Given) is particularly aimed at the quintessential abstract objects, numbers. His distinctions are between matter which is 'individual' and matter which is 'common', and then between entities which are 'sensible' and entities which are 'intelligible'. We can then abstract from these base notions within four permutations, so that we can generate concepts of the 'individual sensible', the 'common sensible', the 'individual intelligible' and the 'common intelligible'. The first three, it is said, can give rise by abstraction to our concepts of natural numbers, because each involves an underlying substance, which gives the necessary units, with the 'common intelligible' category unable to reduce to units when abstraction is applied to it. The 'common intelligible' category is clearly more 'highly' abstract that the 'sensible particular' class, but Aquinas seems not to be suggesting that the former is derived from the latter, in a straightforward sequential derivation, but that the four foundational combinations are part of The Given, leading to a more complex picture when repeated abstractions are applied.

Approaching the levels of abstraction through their psychological origin raises two interesting question about the extremes of the abstraction hierarchy: whether abstraction is already present at the lowest level (the question of whether the Given is innately conceptual), and the metaphysically exciting (but dangerous) question of whether there is a *highest* level, and what it might be like. The nature of the highest level is well beyond the scope of this paper (though William of Ockham hoped to understand the wisdom of God in this way (c.1335:112)), but focus on the bottom of the hierarchy raises the well-known problem of the abstract/concrete borderline. It has become a cliché of modern philosophy to say that this border is hopelessly indeterminate, and might even need to be dropped. When Lewis, for example, addresses the question of whether his real possible worlds should be labelled as 'abstract' or 'concrete', and concludes that they are probably concrete, he nevertheless says that his conclusion is "very ambiguous indeed" (1986:§1.7). The difficulty is, as Fine points out (2005b:14), that possible ways for things to be seems to be an abstraction, while the actual world is a concrete entity, and yet the actual world is also a possible way for things to be; it is almost impossible to pull 'concrete' and 'abstract' apart when we contemplate what exists. Frege seemed to need the distinction, to identify his supply of abstract objects when the concrete objects ran out, but his approach leads to the problem of the hyperinflation of concepts, and Dummett concludes that it does not necessitate the drawing of a sharp line between the concrete and the abstract (1973:494).

Dummett (1973:Ch.14) and Hale (1987:Ch.3) have offered tentative accounts of the borderline, while both admit the difficulties and cite many intractable cases. Dummett investigates the fact that so-called concrete objects can be possible subjects for ostension, while abstract concepts are not. This approaches the ontological question through the empirical epistemology that is involved, and his suggestion is that abstract objects can only be grasped via some function which operates on the physical objects (a role which could be filled by an equivalence principle). Hale notes (p.46) that concrete objects can be perceived, have causal powers, and have location in time and space, but this sort of description invites a flurry of problem cases, such as that an abstract concept such as 'revolution' can have immense causal powers in Czarist Russia, and the 'set' of the children of Henry VIII seems to have had a spatio-temporal location. Plenty of attempts at definition will leave quarks as abstract, because they are so remote from our experience, and many other difficult instances seem to have a definite temporal location, while their spatial location seems uncertain. Other awkward cases which have been offered are a portrait of Napoleon, cuckoo song, the Mistral, space-time points, centres of mass, and patches of colour. Hale's final tentative proposal for a concrete/abstraction distinction (1987:59-61) is based on this fact that abstract objects are held to lack spatial location, while he concedes that they often seem to be temporally located (though his view is at odds with the 'space-time' of modern physics).

Approaching the distinction between concrete and abstract through the creation process of abstractionism will not resolve these difficulties, but it throws some light. Some of the problem cases do not look very difficult. The idea of a 'quark' can be taken as an inference to the best explanation, starting from one set of known physical entities, and ending in another set which is beyond our perceptual powers. The portraits of Napoleon seem to be simple physical objects, with any puzzles about ontological status explained by the concepts and memories in the minds of the audience. The Mistral wind, which Dummett sees as a problem, seems to be an obvious repeated physical process, with the vastness, remoteness and dislocation of the whole object entailing that only an act of mind can encompass it.

The temptation is to pinpoint what is concrete as anything that is part of 'the postulates of physics', and what is abstract as anything which seems to have been derived from the concrete simply by a mental act of ignoring and focusing. But the blurring of the two still seems inescapable when we focus as precisely as possible on the border where they meet, because what we might call 'atoms' of observation (points of pure colour, and so on) seem susceptible to conceptualisation, but there is nothing that can be 'abstracted' from such experiences. Such a problem might encourage the approach of Frege and Geach, that our concepts are creations *concerning* the world, not derivations *from* it. It seems equally plausible, though, that in any account of the actual construction of the human conceptual scheme, such atomic experiences are *not* basic. The account of the Given outlined above sees larger

experiences, of recognition, of generalisation and of complex wholes, as the foundations of our thinking, which might generate concepts like 'red' or 'x strikes y', which can then illuminate the smaller fragments of our experience.

The most intractable cases for the abstractionist approach appear to be the elements of pure mathematics, and the connectives of pure logic, which Frege and Geach (and many others) see as having a necessity which doesn't appear to be derivable from the experience of physical objects. Yablo even offers the fact that abstractions appear to have almost entirely essential properties, and the a priori nature of our grasp of them, as the very hallmark of their distinction from the concrete (2009). However, both Cantor and Dedekind were sympathetic to abstractionist accounts of number, and their views have been defended in modern times by Tait (1996) and Fine (1998). Dedekind thought we could reach a picture of the ordinals by abstracting everything from the natural numbers except their 'distinguishability' and their subjection to the ordering relation (1888:§73). Cantor's suggestion is that The Given for number is an aggregate and an ordering, and that we attain a mental image of numbers by a double act of abstraction from these (see Fine 1998:§1).

Mill is notorious for proposing that numbers can be directly perceived in a handful of pebbles, a view quite reasonably dismissed by Frege, but he also outlined a picture of cross-referenced abstractionism from physical objects which has found sophisticated modern support in the structuralist view of arithmetic (1843: II.vi.2 and III.xxiv.5). Shapiro, a noted champion of the view (1997), happily embraces the abstractionist account: "One way to apprehend a particular structure is through a process of pattern recognition, or abstraction. One observes a system, or several systems with the same structure, and focuses attention on the relations among the objects - ignoring those features of the objects that are not relevant to their relations" (p.74). His favourite illustration is the baseball defence, which can be discussed by ignoring the individual characteristics of the players who implement it (though *not* ignoring the fact that they are human beings who can play baseball). The various structuralist views of mathematics have robust support, and seem to be founded in the human psychological capacity for pattern-recognition (a pattern being a particularly striking resident of the borderline between concrete and abstract).

Among the few writers brave enough to contemplate an abstractionist account of the logical connectives are Russell (1940:Ch.5) and Price (1953:Ch.V). Russell is cautious, and is keen to insist on the separation of the psychological from the logical, but suggests that 'true' might be founded in the feeling that a sentence 'describes' an experience, that 'false' comes from a sense of 'incompatibility', and that 'or' is a "verbal expression of indecision", even felt as "a disjunction in your body". He does wonder, however, whether this psychological 'or' might be something quite distinct from the 'or' of the logician (88).

Price is bolder, and more adamant that our logic is a direct derivation from experience, and he is keen to approach the matter by contemplating animal minds, in order to emphasise how fundamental is the pre-verbal psychological element (part of what he calls 'sign-cognition', as in seeing the coldness of the ice). He takes it as uncontroversial that "disappointed expectation is what brings 'not' into our lives" (124), as when a cat has a dim grasp of the concept 'not yet' when awaiting a saucer of milk. The true concept of 'not' only emerges at the next stage of the abstraction (when 'signs' are experienced), and are part of a "conflict of signs" (127). This is part of the important point made earlier, that abstraction concerns our *experience* of things, not the things themselves; the 'not' is in the cat's mental state (of disappointment), not in the actual absence of the milk. The concept of 'or' is taken as equally unproblematic, as when our cat tries to anticipate the movements of a mouse, to left or to right. The concept of 'if', though, is seen by Price as much more complex, and closer to the heart of logical thought, so he embarks on a heroic discussion of how abstractionism might lead to a pure notion of 'if' (with the cat once again entertaining counterfactual scenarios about the mouse, but with much more complexity arising from this starting point), and hence to genuine logical inference. He recognises that the logicians' 'if' is not found in the minds of animals, but suggests that they are "sensitive to the differences" which we express by conditional language (142).

Modern logic is entirely concerned with internal matters concerning truth, proof and logical consequence, and smiles indulgently on the speculations of Russell and Price, and it must conceded that finding an account of the matter which is both reliable and precise looks a remote possibility, and barely worth pursuing. But that does not make it wrong, and the matter can only be swept under the carpet if we refuse to address the question of where logic fits into our psychology, our epistemology, and our metaphysics.

We will conclude with some remarks about the most famous example in the discussion of abstraction, Frege's derivation of the concept of 'same direction'. The idea is that when two lines are in a relation such as 'parallel', this is sufficient to place them in an equivalence class, and we could then drop the concept of 'parallel', and talk instead of something called the 'direction' of this class of lines (1884:§64), thus leaving the facts untouched, but 'recarving' our conception of them. In nearly all discussions of Frege's example it is assumed that enough work has been done to generate the concept of 'direction', but it is important to note that what he has arrived at is '*same* direction'. One obvious and familiar difficulty to meet here is the matter of co-extensional concepts. Thus, if the given class of parallel lines were all green, and every other line known to us were some other colour, we would be as justified in deriving 'same colour' as in deriving 'same direction', and the class itself could give no explanation of the difference between 'green' and 'direction'. Since a blind person can fully grasp equivalence classes but not 'green', we begin to suspect that something important has been missed. We then note that curved lines, such as railway tracks, can be regarded as parallel, so we will also need the concept of 'straight' (without defining it by reference to 'direction'). And we then realise that the standard notion of 'direction' is a vector concept, unlike 'parallel', so that two different directions (e.g. north-to-south and south-to-north) are conflated within the class that has been labelled 'same direction'.

If we consider taking a second pair of lines which are also parallel to one another, but have a different direction from the first pair, we will once again derive 'same direction' from the new pair by the abstraction principle, but be left

with the problem that the two 'same directions' are actually different. We must now look to a second abstraction, from the two instances of 'same direction', in order to attain the pure concept of 'direction. But we now meet the difficulty that if we try to create an equivalence class between the two instances of 'same direction', in order to derive the univocal concept of 'direction', we find that they are *not* in an equivalence relation (because they are actually *different* directions). An identity between the referents of two singular terms, and hence of some new abstract object, can only be generated if two entities can be in a nameable equivalence relation, which is usually detected in the phrase 'the *same* x' (as when equivalence classes result from being 'the same height', or 'beginning with the same letter'). The only thing which our two classes of lines have in common is a 'direction' of some sort, but that is exactly the concept which we were hoping to create by our procedure. Similar points can be made about Frege's example of the 'same shape' of similar triangles, where our route to deriving 'shape' from triangles and squares is blocked by the fact that there are *different* shapes. We can, of course, apply derelativization to the relation of 'same direction', to assert that a line exists which has 'the direction' of the given line, but this won't facilitate the jump to the innumerable *different* directions which interest us. We seem to have reached an impasse.

There may be a simple solution to this problem, one which saves both approaches. We noted earlier a parallel between the problem of abstractionism and the problem of truth. The Tarskian version of truth is offered as a corrective to the vagueness of traditional accounts, resulting in a wave of enthusiasm for the new approach, followed by a sense of dissatisfaction that something rather important is missing. This essay has suggested that an identical situation faces us with regard to abstract concepts. However, remembering a simple distinction might help: in each case, are we trying to *define* the target concepts, or *understand* them? A central feature of any definition must be that the *definiendum* is unique to the *definiens*, that is, that only one item will fit the proposed definition. Imprecision and ambiguity are therefore anathema to a good definition, and the approach to the definition of abstract concepts by equivalence achieves precisely that (provided that our lines are only related in a single equivalent respect, such as 'parallel').

On the other hand, the aim of traditional accounts of abstraction (and of truth) has always been *understanding*. Consider, for example, the distinction between a definition of the concept of 'evolution', and an understanding of it. The best route to the latter seems to be an understanding of its genesis in the early nineteenth century, as the evidence accumulated. No one could be considered to have grasped the concept of 'direction' if they did not understand the metaphorical notion in the 'direction of an argument', or the fact that two directions could differ from one another, or that a direction can be vague, or have an opposite, or that a curve has a tangential direction at a point. No equivalence principle is ever going to provide such a full account of a concept. Frege seems to have been motivated only by a desire for a definition, though the modern approach *might* also contribute to our understanding of the metaphysics of abstract concept, as part of a metaphysics of the objects which bear properties, constitute classes, and are the subject-matter of quantification. But a full metaphysic needs an understanding of the full range of roles played by concepts, and must dovetail our account of thought into our account of the mind, and here it would be a serious mistake to ignore the insights of abstractionism.

11,963 words

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